Vector calculus - Test 1

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Parametric foundations and motion.

A scientific calculator may be used.

1. Consider the parametric curve $x = t + 1, y = t^2$, $0 \le t \le 2$. [6 marks]

(a) Eliminate the parameter and state the traced portion. [3 marks]

(b) Describe its orientation using endpoints and any repeated tracing. [3 marks]

2. In metres at time t seconds, A is at $(1 + 2t, 3 - t)$ and B at $(7 - t, t/2)$, for $t \ge 0$. [6 marks]

(a) Find whether and when they collide. [3 marks]

(b) Find their speeds. [3 marks]

3. $\mathbf{r} = (t, 6t - t^2)$, $0 \leq t \leq 6$. [6 marks]

(a) Find the greatest height and the velocity there. [3 marks]

(b) Can the particle ever stop? Explain. [3 marks]

4. A quadratic Bézier path has controls (0, 0), (2, 4), (6, 0). Use the standard Bézier parameter interval $0 \leq t \leq 1$. [6 marks]

(a) Find B (1/2). [3 marks]

(b) Find both endpoint derivative vectors. [3 marks]

5. $\mathbf{v} = (2t, 3)$, $\mathbf{r}(0) = (1, -2)$. [6 marks]

(a) Find the position function. [3 marks]

(b) Find displacement from 0 to 2. [3 marks]

6. $\mathbf{r} = (t^2, 2t)$. [6 marks]

(a) Find velocity and acceleration. [3 marks]

(b) Find speed and speed rate at 1. [3 marks]

7. For $x = t^2, y = t^3$, t=1. [6 marks]

(a) Find the point and tangent gradient at $t = 1$. [3 marks]

(b) Give the tangent line. [3 marks]

8. $\mathbf{r} = (2\cos 3t, 2\sin 3t)$. [6 marks]

(a) Describe the locus and find its period. [3 marks]

(b) Find speed and acceleration magnitude. [3 marks]

9. $\mathbf{r} = (3t, 4t), 0 \leq t \leq 2$. [6 marks]

(a) Set up the arc-length integral. [3 marks]

(b) Evaluate the exact curve length. [3 marks]

10. A camera follows $\mathbf{r} = (t, 4t - t^2)$ metres, $0 \leq t \leq 4$. [6 marks]

(a) Find initial/final positions and the greatest height. [3 marks]

(b) Find velocity at the greatest height and explain why the camera is still moving. [3 marks]