Vector calculus - Test 2

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Trace restrictions, branches and repeated paths.

A scientific calculator may be used.

1. Consider the parametric curve $x = t^2, y = 2t, -1 \leq t \leq 1$. [6 marks]

(a) Eliminate the parameter and state the traced portion. [3 marks]

(b) Describe its orientation using endpoints and any repeated tracing. [3 marks]

2. A is at (t, t) and B at $(2 - t, 3 - t)$, for $t \geq 0$. [6 marks]

(a) Show that there is no collision. [3 marks]

(b) Find the distance between them when their x coordinates agree. [3 marks]

3. $\mathbf{r} = (t^2, t^3 - t)$, $-1 \leq t \leq 1$. [6 marks]

(a) Find and verify a repeated point. [3 marks]

(b) Find velocity at t=0 and decide whether it is at rest. [3 marks]

4. A cubic Bézier path has controls (0, 0), (1, 2), (3, 2), (4, 0). Use the standard Bézier parameter interval $0 \leq t \leq 1$. [6 marks]

(a) Find the start and end positions. [3 marks]

(b) Find both endpoint derivatives and explain the tangent directions. [3 marks]

5. $\mathbf{a} = (6t, 2), \mathbf{v}(0) = (1, -1), \mathbf{r}(0) = (0, 3)$. [6 marks]

(a) Find the position function. [3 marks]

(b) Find position and velocity at t=1. [3 marks]

6. $\mathbf{r} = (3\cos t, 3\sin t)$. [6 marks]

(a) Find velocity and acceleration. [3 marks]

(b) Find speed and acceleration magnitude and explain whether speed changes. [3 marks]

7. For $x = t^2, y = t$, t=0. [6 marks]

(a) Find the point and derivative vector at $t = 0$. What does the vector show about the tangent? [3 marks]

(b) Give the tangent line. [3 marks]

8. $\mathbf{r} = (3\cos t, 2\sin t)$. [6 marks]

(a) Describe the locus and find its period. [3 marks]

(b) Compare speeds at 0 and $\pi/2$; is speed constant? [3 marks]

9. $\mathbf{r} = (t^2, \frac{2}{3}t^3), 0 \leq t \leq 1$. [6 marks]

(a) Set up the arc-length integral. [3 marks]

(b) Evaluate the exact curve length. [3 marks]

10. A robot follows $\mathbf{r} = (\cos t, \sin t), 0 \leq t \leq 4\pi$. [6 marks]

(a) Find its total distance and displacement. [3 marks]

(b) Explain what is lost if the journey is recorded only as x squared+y squared=1. [3 marks]