Vector calculus - Test 3

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Polynomial, Bezier and vector applications.

A scientific calculator may be used.

1. Consider the parametric curve $x = 2\cos t, y = \sin t$, $0 \le t \le \pi$. [6 marks]

(a) Eliminate the parameter and state the traced portion. [3 marks]

(b) Describe its orientation using endpoints and any repeated tracing. [3 marks]

2. A particle moves uniformly from (1, 2, 3) m to (7, -2, 3) m in 2 s. [6 marks]

(a) Write its position function. [3 marks]

(b) Find speed and total distance over the two seconds. [3 marks]

3. $\mathbf{r} = (t^2 - 2t, t^3 - 3t)$, $0 \leq t \leq 2$. [6 marks]

(a) Find when the particle is at rest and its position. [3 marks]

(b) Find its acceleration at that instant. [3 marks]

4. A quadratic Bézier path uses (0, 0), (2, 3), (4, 0). Use the standard Bézier parameter interval $0 \leq t \leq 1$. [6 marks]

(a) Expand its coordinate rules. [3 marks]

(b) Find the greatest y and explain whether the path passes through the middle control. [3 marks]

5. $\mathbf{v} = (e^t, \cos t), \mathbf{r}(0) = (2, 0)$. [6 marks]

(a) Find the position function. [3 marks]

(b) Find displacement from 0 to $\pi/2$. [3 marks]

6. $\mathbf{r} = (e^t, e^{-t})$. [6 marks]

(a) Find velocity and acceleration. [3 marks]

(b) Find speed and speed rate at 0. [3 marks]

7. For $x = \cos t, y = \sin t$, t=0. [6 marks]

(a) Find the point and tangent direction at $t = 0$. [3 marks]

(b) Give the tangent line. [3 marks]

8. $\mathbf{r} = (1 + 4\cos 2t,\ -2 - 4\sin 2t)$. [6 marks]

(a) Describe the locus and find its period. [3 marks]

(b) Find direction and speed. [3 marks]

9. $\mathbf{r} = (\cos t, \sin t, 2t), 0 \leq t \leq 2\pi$. [6 marks]

(a) Set up the arc-length integral. [3 marks]

(b) Evaluate the exact curve length. [3 marks]

10. A drawing tool follows a quadratic Bézier curve with controls (0, 0), (2, 2), (4, 0). Use the standard Bézier parameter interval $0 \leq t \leq 1$. [6 marks]

(a) Expand its coordinate rules and find the highest point. [3 marks]

(b) Find the speed at t=1/2 and state whether the tool stops. [3 marks]