Vector calculus - Test 4

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Tangent directions and three-dimensional travel.

A scientific calculator may be used.

1. Consider the parametric curve $x = 3 - t, y = (t - 1)^2$, $0 \leq t \leq 3$. [6 marks]

(a) Eliminate the parameter and state the traced portion. [3 marks]

(b) Describe its orientation using endpoints and any repeated tracing. [3 marks]

2. A runner has position $(2t, 0)$ m and a cyclist starting 10 m behind has position $(-10 + 4t, 0)$ m, for $t \geq 0$. [6 marks]

(a) Find catch-up time and position. [3 marks]

(b) Explain why equal position at that instant does not mean equal velocity. [3 marks]

3. $\mathbf{r} = (t^2, 4t)$, $0 \leq t \leq 2$. [6 marks]

(a) Eliminate t and state the restriction. [3 marks]

(b) Find velocity and decide whether rest is possible. [3 marks]

4. A quadratic path ends with controls P 1=(1, 2), P 2=(3, 1). A second quadratic begins with Q 0=P 2 and unknown Q 1. [6 marks]

(a) Find the first path final derivative. [3 marks]

(b) Find Q 1 so the two derivatives are equal at their join. [3 marks]

5. $\mathbf{v} = (3t^2, 2t, 1), \mathbf{r}(0) = (0, 1, -1)$. [6 marks]

(a) Find the position function. [3 marks]

(b) Find displacement from 0 to 1. [3 marks]

6. $\mathbf{r} = (3t, 4t, 12t)$. [6 marks]

(a) Find velocity and acceleration. [3 marks]

(b) Find speed and explain its constancy. [3 marks]

7. For $\mathbf{r} = (t, t^2, t^3)$, t=1. [6 marks]

(a) Find the point and derivative vector at $t = 1$. [3 marks]

(b) Give a vector equation of the tangent line, using a new parameter. [3 marks]

8. $\mathbf{r} = (3\cos t, 3\sin t, 4t)$. [6 marks]

(a) Describe the helix radius and find the time for one planar turn. Does the full position function have a period? [3 marks]

(b) Find speed and rise per turn. [3 marks]

9. $\mathbf{r} = (4\cos t, 4\sin t), 0 \le t \le \pi/2$. [6 marks]

(a) Set up the arc-length integral. [3 marks]

(b) Evaluate the exact curve length. [3 marks]

10. A helix is $\mathbf{r} = (2\cos t, 2\sin t, 3t)$, from 0 to π. [6 marks]

(a) Find the endpoint separation. [3 marks]

(b) Find path length and explain why it exceeds the separation. [3 marks]