Vector calculus - Test 5

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Reasoning about speed, rest and numerical arc length.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Consider the parametric curve $x = t^2, y = t^2 + 1, -2 \le t \le 2$. [6 marks]

(a) Eliminate the parameter and state the traced portion. [3 marks]

(b) Describe its orientation using endpoints and any repeated tracing. [3 marks]

2. A moves as $(t, 0)$ and B as $(0, 2 - t)$, in metres for $t \ge 0$. [6 marks]

(a) Write squared separation as a function of t. [3 marks]

(b) Find closest separation and its time. [3 marks]

3. $\mathbf{r} = (t^3 - 3t, t^2)$, $-2 \leq t \leq 2$. [6 marks]

(a) Find the x-coordinate turning parameters. [3 marks]

(b) Find velocity and tangent orientation at those parameters. [3 marks]

4. A quadratic Bézier curve has P 0=(0, 0), P 2=(4, 0), and initial derivative (2, 6). Use the standard Bézier parameter interval $0 \leq t \leq 1$. [6 marks]

(a) Determine its middle control P 1. [3 marks]

(b) Find B (1/2). [3 marks]

5. $\mathbf{r} = (t^3, e^{2t}, \sin 3t)$. [6 marks]

(a) Find the velocity function. [3 marks]

(b) Find acceleration. [3 marks]

6. $\mathbf{r} = (2\cos t, 2\sin t, 3t)$. [6 marks]

(a) Find velocity and acceleration. [3 marks]

(b) Find speed and explain its constancy. [3 marks]

7. For $x = t^3, y = t^2$, t=0. [6 marks]

(a) Find the point and explain why direct substitution into the derivative ratio is inconclusive. [3 marks]

(b) Use a limiting direction to find the tangent line. [3 marks]

8. $\mathbf{r} = (2\cos(t + \pi/2), 2\sin(t + \pi/2))$. [6 marks]

(a) Describe the locus and find its period. [3 marks]

(b) Find initial velocity and orientation. [3 marks]

9. $\mathbf{r} = (t, t^2), 0 \leq t \leq 1$. [6 marks]

(a) Set up the arc-length integral. [3 marks]

(b) Use numerical integration to find the curve length to five decimal places. [3 marks]

10. A particle has $\mathbf{r} = (t^2, t^3 - t)$ for $-1 \leq t \leq 1$. [6 marks]

(a) Show that its endpoints coincide but it is not stationary at t=0. [3 marks]

(b) Set up an integral for the full travelled distance. [3 marks]