Vector calculus - Test 6

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

Joined curves and combined parametric reasoning.

A scientific calculator may be used.

1. Consider the parametric curve $x = 1 + 2t, y = 4 - t$, $1 \leq t \leq 4$. [6 marks]

(a) Eliminate the parameter and state the traced portion. [3 marks]

(b) Describe its orientation using endpoints and any repeated tracing. [3 marks]

2. A boat has velocity (3, 0) m/s relative to water, while water flows with velocity (0, 1) m/s. It starts at the origin and the opposite bank is x=30 m. [6 marks]

(a) Find its ground-position function. [3 marks]

(b) Find arrival time, downstream drift and ground distance. [3 marks]

3. $\mathbf{r} = (2t - t^2, t^2)$, $0 \leq t \leq 2$. [6 marks]

(a) Find the rightmost point. [3 marks]

(b) Find velocity and speed there. [3 marks]

4. A quadratic Bézier curve has endpoints P 0=(0, 0), P 2=(6, 0) and B (1/2)=(2.5, 2). Use the standard Bézier parameter interval $0 \leq t \leq 1$. [6 marks]

(a) Determine the middle control. [3 marks]

(b) Find its endpoint derivatives. [3 marks]

5. $\mathbf{v} = (\cos t, \sin t, 2), \mathbf{r}(0) = (0, 0, 1)$. [6 marks]

(a) Find the position function. [3 marks]

(b) Find displacement from 0 to π. [3 marks]

6. $\mathbf{r} = (t^2 - 2t, t^3 - 3t)$. [6 marks]

(a) Find velocity and acceleration. [3 marks]

(b) Find rest times and acceleration there. [3 marks]

7. For $x = 2t, y = t^2 - 4t$, t=2. [6 marks]

(a) Find the point and tangent gradient at $t = 2$. [3 marks]

(b) Give the tangent line. [3 marks]

8. $\mathbf{r} = (5\cos(2t), 5\sin(2t), -t)$. [6 marks]

(a) Describe the helix radius and find the time for one planar turn. Does the full position function have a period? [3 marks]

(b) Find speed and vertical change per turn. [3 marks]

9. $\mathbf{r} = (2\cos t, 2\sin t), 0 \le t \le 4\pi$. [6 marks]

(a) Set up the arc-length integral. [3 marks]

(b) Evaluate the exact curve length. [3 marks]

10. A quadratic Bézier path has controls P 0=(0, 0), P 1=(1, 2), P 2=(3, 2). A cubic begins at Q 0=P 2, with Q 1 unknown. Use the standard Bézier parameter interval $0 \leq t \leq 1$. [6 marks]

(a) Find the quadratic final derivative. [3 marks]

(b) Find Q 1 so the cubic has the same initial derivative, not merely a parallel tangent. [3 marks]