Zeros, roots and factors

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. List the zeros of $(x-5)(x+2)(2x-3)$. [2 marks]

Set each factor equal to zero.

Answer: $5,\ -2,\ \frac{3}{2}$

2. State multiplicities and crossing behaviour for $x^2(x+1)^3$. [2 marks]

Even powers preserve sign; odd powers change sign.

Answer: 0 has multiplicity 2 and touches; -1 has multiplicity 3 and crosses.

3. Check whether -1 is a zero of x^3+2x^2-x-2. [2 marks]

$P(-1) = -1+2+1-2 = 0$.

Answer: Yes.

4. Write a monic cubic with zeros $-2, 1, 4$. [2 marks]

The linear factors give each root; monic means leading coefficient 1.

Answer: $(x+2)(x-1)(x-4) = x^3-3x^2-6x+8$

5. Give a quadratic with no real zeros and explain how you know. [2 marks]

For real x, $x^2 \geq 0$, so $x^2+1>0$.

Answer: For example x^2+1.

6. A quartic has three distinct zeros, one of them repeated twice and the others simple. How many roots are counted with multiplicity? [2 marks]

$2+1+1=4$. The repeated zero is still one distinct input.

Answer: Four.