Polynomial identities and coefficient comparison

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Find a, b if $(x+1)(x+a) \equiv x^2 + 5x + b$. [2 marks]

Expansion is $x^2 + (a+1)x + a$. Compare the x and constant coefficients.

Answer: $a = 4, b = 4$

2. Find a, b if $(2x-1)(x^2 + ax + b) \equiv 2x^3 + 3x^2 - 4x + 1$. [2 marks]

$2a - 1 = 3$ gives $a = 2$; $-b = 1$ gives $b = -1$; then $2b - a = -4$.

Answer: $a = 2, b = -1$

3. Find A, B if $5x - 1 \equiv A(x+1) + B(x-2)$. [2 marks]

Set $x = 2$ to get $9 = 3A$; set $x = -1$ to get $-6 = -3B$.

Answer: $A = 3, B = 2$

4. A student claims $x^3 - x \equiv 0$ because it is zero at $-1, 0, 1$. Explain the error. [2 marks]

For example, at $x = 2$ the expression equals 6.

Answer: Three zeros are possible for a non-zero cubic; this is not an identity.

5. Find a, b, c if $ax^2 + bx + c \equiv (x-2)^2 + 3(x+1)$. [2 marks]

Expand: $x^2 - 4x + 4 + 3x + 3 = x^2 - x + 7$.

Answer: $a = 1, b = -1, c = 7$

6. Two polynomials of degree at most 2 agree at three distinct inputs. Must they be identical? Explain. [3 marks]

Their difference has degree at most 2 and has three distinct zeros. Only the zero polynomial can do this.

Answer: Yes.