Finding and factorising real polynomial roots

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Factor $x^3 - 7x + 6$. [2 marks]

Test 1, divide to obtain $x^2 + x - 6$, then factor.

Answer: $(x - 1)(x - 2)(x + 3)$

2. Factor $x^3 + x^2 - 4x - 4$. [2 marks]

Grouping gives $(x + 1)(x^2 - 4)$.

Answer: $(x + 1)(x - 2)(x + 2)$

3. Factor $2x^3 - 3x^2 - 8x + 12$. [2 marks]

Grouping gives $(2x - 3)(x^2 - 4)$.

Answer: $(2x - 3)(x - 2)(x + 2)$

4. Factor $x^4 - 5x^2 + 4$ and state all its real roots. [2 marks]

Treat the expression as a quadratic in x^2: $(x^2 - 1)(x^2 - 4)$.

Answer: $(x - 1)(x + 1)(x - 2)(x + 2)$; roots $\pm 1, \pm 2$.

5. Factor $x^3 - 3x^2 - 4x + 12$. [2 marks]

Grouping gives $(x - 3)(x^2 - 4)$.

Answer: $(x - 3)(x - 2)(x + 2)$

6. Factor $x^4 - 1$ as far as possible over the reals. Why does one quadratic remain? [3 marks]

Use difference of squares twice. The quadratic $x^2 + 1$ has negative discriminant, so has no real linear factors.

Answer: $(x - 1)(x + 1)(x^2 + 1)$