Graphing real polynomials

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. For $P(x) = x(x-2)^2$, state roots, multiplicities and end behaviour. [2 marks]

The leading term is x^3 and the factor powers give multiplicities.

Answer: 0 simple/crosses; 2 double/touches; left down, right up.

2. Find the y-intercept and end behaviour of $P(x) = -2(x+1)(x-3)$. [2 marks]

$P(0) = -2(1)(-3) = 6$; leading term $-2x^2$.

Answer: Y-intercept $(0, 6)$; both ends down.

3. State the sign of $(x+1)^2(x-2)$ on the intervals separated by -1 and 2. [2 marks]

The square is positive except at -1, so the other factor determines the sign.

Answer: Negative on $(-\infty, -1)$ and $(-1, 2)$; positive on $(2, \infty)$.

4. Write a monic quartic that touches the axis at -2 and 1 and has no other roots. Find its y-intercept. [2 marks]

Each touching root requires an even multiplicity; two double roots use degree four.

Answer: $P(x) = (x+2)^2(x-1)^2$; y-intercept $(0, 4)$.

5. Find the stationary points of $P(x) = x^3 - 6x^2 + 9x$. [4 marks]

$P' = 3(x-1)(x-3)$, so candidates are $1, 3$. $P''' = 6x - 12$ determines the types.

Answer: $(1, 4)$ local maximum and $(3, 0)$ local minimum.

6. Explain why a positive-leading cubic must have at least one real zero. [3 marks]

Polynomial graphs are continuous, so a change from negative to positive cannot occur without taking the value zero.

Answer: Its far-left values are negative and far-right values positive; continuity forces a crossing.