Solving real polynomial equations

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $x^3 - 7x + 6 = 0$. [2 marks]

Factor as $(x + 3)(x - 1)(x - 2) = 0$.

Answer: $x = -3, 1, 2$

2. Solve $x^4 - 10x^2 + 9 = 0$. [2 marks]

Put $u = x^2$; $(u - 1)(u - 9) = 0$.

Answer: $x = \pm 1, \pm 3$

3. Solve $x^4 + x^2 - 6 = 0$ over the reals. [2 marks]

$(x^2 + 3)(x^2 - 2) = 0$; the first factor has no real zero.

Answer: $x = \pm\sqrt{2}$

4. Solve $2x^3 - 3x^2 - 8x + 12 = 0$. [2 marks]

Factor as $(2x - 3)(x - 2)(x + 2) = 0$.

Answer: $x = -2, \frac{3}{2}, 2$

5. A model gives $t(t - 2)(t + 5) = 0$ for a time with $0 < t < 4$. State the allowed solution and explain both exclusions. [2 marks]

$t = 0$ is excluded by the strict lower bound; $t = -5$ is negative and outside the domain.

Answer: $t = 2$ only.

6. A student solves $x^3 = 9x$ by dividing by x and reports $x = \pm 3$. Correct the answer. [3 marks]

The original equation is also true at $x = 0$. Write $x(x - 3)(x + 3) = 0$ to keep all roots.

Answer: $x = -3, 0, 3$