Real Polynomials - Practice Test 1

Name: _____ Date: _____

10 questions | 43 marks | 65 minutes

Build from polynomial operations to factor theorems, graphs and a volume application.

A scientific calculator may be used.

1. Consider $P(x) = 3x^4 - 2x^3 + 7x - 5$. [3 marks]

(a) State its degree and leading coefficient. [1 mark]

(b) Expand $(x - 2)(x^2 + x + 3)$. [2 marks]

2. Let $Q(x) = (x + 3)^2(2x - 1)$. [4 marks]

(a) State its real roots and their multiplicities. [2 marks]

(b) State whether the graph crosses or touches at each root. [2 marks]

3. The identity $(x+1)(ax^2+bx+c) \equiv 2x^3+5x^2+4x+1$ holds. [4 marks]

(a) Find a, b, c. [3 marks]

(b) Explain why substituting one value would not be enough to prove the identity. [1 mark]

4. Divide x^3-2x^2-5x+6 by $x-1$. [4 marks]

(a) Find the quotient and remainder. [3 marks]

(b) Hence factor the cubic completely. [1 mark]

5. Let $P(x) = 2x^3 - 3x^2 + 1$. [4 marks]

(a) Find the remainder on division by $x + 1$. [2 marks]

(b) Find the remainder on division by $2x - 1$. [2 marks]

6. $P(x) = x^3 + kx^2 - 4x - 4$ has factor $x + 1$. [5 marks]

(a) Find k. [2 marks]

(b) Find all real roots. [3 marks]

7. Consider $R(x) = x^4 - 5x^2 + 4$. [4 marks]

(a) Factor R using a substitution. [2 marks]

(b) Solve $R(x) = 0$ over the reals. [2 marks]

8. Sketch $y = (x + 1)^2(x - 2)$ without a graphics calculator. [5 marks]

(a) State all intercepts and root multiplicities. [3 marks]

(b) Describe the two ends and the behaviour at each intercept. [2 marks]

9. A student cancels x from $x^3 = 4x$ and reports two roots. [4 marks]

(a) Explain which solution is lost. [2 marks]

(b) Solve correctly. [2 marks]

10. Squares of side x cm are cut from the corners of a 10 cm by 8 cm sheet and the sides are folded into an open box. [6 marks]

(a) Write a volume model and its physical domain. [3 marks]

(b) For $V = 48$ cm^3, use $x^3 - 9x^2 + 20x - 12 = (x-1)(x-2)(x-6)$ to find the possible cuts. [3 marks]