Real Polynomials - Practice Test 2

Name: _____ Date: _____

10 questions | 45 marks | 65 minutes

Use identities, remainders and repeated roots to reason about polynomial structure.

A scientific calculator may be used.

1. Classify and simplify carefully. [4 marks]

(a) Which are polynomials in x: $x^3 + \sqrt{2}$, $x^{-1} + 4$, $(x^2 - 1)/(x - 1)$ as originally written? [3 marks]

(b) Simplify $(2x^2 + 3x - 1) + (x^3 - 5x + 4)$. [1 mark]

2. A cubic has zeros $-1, 2, 3$ and $P(0) = 12$. [4 marks]

(a) Find its factorised form. [3 marks]

(b) Find its leading coefficient and y-intercept. [1 mark]

3. $P(x) = ax^2 + bx + c$ satisfies $P(0) = 2$, $P(1) = 6$, $P(-1) = 4$. [5 marks]

(a) Find a, b, c. [3 marks]

(b) Explain why these conditions determine a unique polynomial of degree at most 2. [2 marks]

4. A division gives $D(x) = x^2 + 1$, quotient $Q(x) = 2x - 3$ and remainder $R(x) = x + 4$. [3 marks]

(a) Reconstruct the dividend. [2 marks]

(b) Explain why the stated remainder has an allowable degree. [1 mark]

5. $P(2) = 7$ and $P(-1) = 1$. [5 marks]

(a) Find the remainder when P is divided by $(x-2)(x+1)$. [3 marks]

(b) Can these data determine the whole polynomial? Explain. [2 marks]

6. Let $P(x) = x^3 - 3x^2 + 4$. [5 marks]

(a) Show that 2 is a repeated root. [3 marks]

(b) State the sign of P immediately on either side of 2. [2 marks]

7. Factor $x^3 + x^2 + x + 1$. [4 marks]

(a) Give a complete real factorisation. [2 marks]

(b) State all real roots and explain why the quadratic gives none. [2 marks]

8. For $f(x) = x^3 - 3x$, use calculus to refine the sketch. [6 marks]

(a) Find the stationary points. [3 marks]

(b) Classify them and state the real zeros. [3 marks]

9. Solve $x^4 + x^2 - 6 = 0$ over the reals. [4 marks]

(a) Use a substitution to factor. [2 marks]

(b) Explain why there are only two real solutions. [2 marks]

10. Consider $P_c(x) = (x-1)^2 + c$. [5 marks]

(a) Find the real roots in terms of c when they exist. [3 marks]

(b) State the number of distinct real roots for $c < 0$, $c = 0$ and $c > 0$. [2 marks]