Real Polynomials - Practice Test 3

Name: _____ Date: _____

10 questions | 51 marks | 65 minutes

Apply polynomial models to geometry, intersections and reconstruction problems.

A scientific calculator may be used.

1. Let $P(x) = x^3 + 2x^2 - x - 2$. [4 marks]

(a) Find $P(0)$ and $P(1)$. [1 mark]

(b) Factor P completely and list its real roots. [3 marks]

2. A rectangle has side lengths $x + 2$ and $x + 5$ metres, where $x > 0$. [5 marks]

(a) Expand its area rule. [2 marks]

(b) Find x when the area is 28 m^2. [3 marks]

3. A monic cubic $P(x) = x^3 + ax^2 + bx + 2$ has zeros 1 and 2. [5 marks]

(a) Find a and b. [3 marks]

(b) Find the remaining root. [2 marks]

4. For $P(x) = 2x^3 + 3x^2 - 8x + 3$, consider the proposed factor $2x - 1$. [5 marks]

(a) Prove it is a factor. [2 marks]

(b) Find the full real factorisation. [3 marks]

5. The curve $y = x^3 - 4x$ meets the line $y = 2x$. [5 marks]

(a) Form and solve the equation for their x-coordinates. [3 marks]

(b) State all intersection points. [2 marks]

6. A student solves $(x - 1)(x^2 - 4) = 0$ by cancelling $x - 1$. [4 marks]

(a) Explain the error. [2 marks]

(b) Give the complete solution set. [2 marks]

7. $P(x) = (x+1)(x-2)$ and $Q(x) = -3P(x-1)$. [4 marks]

(a) Write Q in factorised form and find its zeros. [3 marks]

(b) Describe the end behaviour of Q. [1 mark]

8. Consider $f(x) = x^3 - 3x^2 + 2$. [7 marks]

(a) Find and classify the stationary points. [4 marks]

(b) Show that its three roots are $1, 1-\sqrt{3}, 1+\sqrt{3}$. [3 marks]

9. An open box is formed by cutting squares of side x cm from a 12 cm by 6 cm sheet. [6 marks]

(a) Form the volume model and physical domain. [2 marks]

(b) For volume 40 cm^3, use $x^3 - 9x^2 + 18x - 10 = (x - 1)(x^2 - 8x + 10)$ to find all possible cuts. [4 marks]

10. Design a real polynomial graph of the least possible degree that crosses at -1 and 1, touches at 2, and passes through (0,-8). [6 marks]

(a) Find a factorised rule. [4 marks]

(b) State its degree, leading coefficient and end behaviour. [2 marks]