Real Polynomials - Practice Test 4

Name: _____ Date: _____

10 questions | 45 marks | 65 minutes

Explain why polynomial methods work, and diagnose claims that need stronger justification.

A scientific calculator may be used.

1. Let P be a non-zero cubic and Q a non-zero quadratic. [3 marks]

(a) State the degree of PQ. [1 mark]

(b) Can $3P - 4Q$ have degree 2? Explain. [2 marks]

2. A student tests two polynomials of degree at most 4 at four distinct inputs and finds equal outputs. [5 marks]

(a) Explain why identity has not yet been proved. [2 marks]

(b) Give an example of such a difference, and state how many distinct matching inputs would suffice. [3 marks]

3. $P(1) = 2$ and $P(-1) = 4$. [5 marks]

(a) Find its remainder on division by $x^2 - 1$. [3 marks]

(b) Explain why $P(x) + x - 3$ is divisible by $x^2 - 1$. [2 marks]

4. A monic cubic has a double root at 1 and a simple root at -2. [4 marks]

(a) Find its expanded rule. [3 marks]

(b) State the graph behaviour at both roots. [1 mark]

5. Let $P(x) = x^3 - 4x^2 + 5x - 2$. [5 marks]

(a) Show that 1 has multiplicity at least 2. [3 marks]

(b) Why is its multiplicity exactly 2? [2 marks]

6. Consider $P(x) = x^4 + 4$. [5 marks]

(a) Verify $P = (x^2 - 2x + 2)(x^2 + 2x + 2)$. [3 marks]

(b) Explain why P has no real zero. [2 marks]

7. $P(x) = (x-2)(x-1)(x+1)$. A student claims that the solutions of $P(x) = 5$ are $2, 1, -1$. [3 marks]

(a) Refute the claim using a substitution. [1 mark]

(b) Explain which form an equation needs before the zero-product principle applies. [2 marks]

8. A non-zero quartic is claimed to have five distinct real roots. [4 marks]

(a) Explain the contradiction. [2 marks]

(b) Could a quartic have one real root with multiplicity 4? Give an example. [2 marks]

9. $P_k(x) = x^3 + kx^2 - x - k$. [6 marks]

(a) Factor and find its real roots in terms of k. [3 marks]

(b) For which real k are there only two distinct roots? [3 marks]

10. Two models are $P(x) = x^2$ and $Q(x) = x^2 + (x-1)(x-2)(x-3)$. [5 marks]

(a) Explain why observations at x=1,2,3 cannot distinguish them. [2 marks]

(b) Find their predictions at x=4 and explain the lesson for modelling. [3 marks]