Real Polynomials - Practice Test 5

10 questions | 49 marks | 75 minutes

Connect exact algebra, restricted domains and numerical evidence without losing solutions.

Equivalent correct methods earn the corresponding marks.

1. Consider $F(x) = \frac{x^2-4}{x-2}$. [4 marks]

(a) Simplify and state the original domain. [2 marks]

Factor the numerator and cancel only where $x - 2 \neq 0$.

Answer: $F(x) = x + 2$ for $x \neq 2$.

(b) Is F the same function as the polynomial $x + 2$ on all real inputs? [2 marks]

Equal formulas on a restricted domain do not erase the original hole.

Answer: No; F is undefined at 2.

2. $P(x) = 2x^3 - 3x^2 - 11x + 6$. [5 marks]

(a) Show that $\frac{1}{2}$ is a root. [2 marks]

$\frac{1}{4} - \frac{3}{4} - \frac{11}{2} + 6 = 0$.

Answer: $P(\frac{1}{2}) = 0$.

(b) Factor P completely. [3 marks]

Divide by $2x - 1$ to get $x^2 - x - 6$, then factor.

Answer: $(2x - 1)(x + 2)(x - 3)$.

3. An odd cubic has rule $P(x) = ax^3 + bx$ and satisfies $P(1) = 2$, $P(2) = 10$. [5 marks]

(a) Find a and b. [3 marks]

$a + b = 2$, $4a + b = 5$; subtract to find $a = 1$, then $b = 1$.

Answer: $a = 1, b = 1$.

(b) Find all real zeros and explain why no others exist. [2 marks]

$P = x(x^2 + 1)$ and $x^2 + 1 > 0$ on the real line.

Answer: $x = 0$ only.

4. Divide $P(x) = 6x^3 - x^2 - 8x + 4$ by $2x - 1$. [5 marks]

(a) Find Q and R. [3 marks]

Long division successively gives $3x^2$, x, $-\frac{7}{2}$, leaving $\frac{1}{2}$.

Answer: $Q = 3x^2 + x - \frac{7}{2}$, $R = \frac{1}{2}$.

(b) Check the remainder by the remainder theorem. [2 marks]

$6(\frac{1}{8}) - \frac{1}{4} - 4 + 4 = \frac{1}{2}$.

Answer: $P(\frac{1}{2}) = \frac{1}{2}$.

5. $P(2) = 5$ and $P(-2) = 1$. [4 marks]

(a) Find the remainder on division by $x^2 - 4$. [3 marks]

$2m + c = 5$ and $-2m + c = 1$ give $m = 1, c = 3$.

Answer: $R(x) = x + 3$.

(b) Find the remainder of $P(x) - x - 3$ on the same division. [1 mark]

The added terms cancel the linear remainder.

Answer: 0.

6. $P(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$. It is known that $(x-2)^2$ is a factor. [5 marks]

(a) Find the remaining quadratic factor. [3 marks]

Division, or coefficient comparison, gives $P = (x-2)^2(x^2 + 2x + 1)$.

Answer: $x^2 + 2x + 1 = (x+1)^2$.

(b) State the real roots, multiplicities and sign of P elsewhere. [2 marks]

P is a product of two squares and vanishes only at those two inputs.

Answer: -1 and 2 are double; $P > 0$ elsewhere.

7. Solve $\frac{x^3 - x}{x-1} = 6$. [5 marks]

(a) State the restriction and form the polynomial equation obtained by multiplication. [2 marks]

Multiply by $x - 1$ only after recording its non-zero restriction.

Answer: $x \neq 1$; $x^3 - 7x + 6 = 0$.

(b) Find the allowed solutions. [3 marks]

The multiplied equation factors as $(x-1)(x-2)(x+3) = 0$; exclude the root 1 from the original domain.

Answer: $x = -3, 2$.

8. For $f(x) = x^3 - x - 1$, numerical calculations give $f(1.3) = -0.103$, $f(1.4) = 0.344$, $f(1.32) = -0.020032$ and $f(1.325) = 0.001203125$. [5 marks]

(a) Give a justified interval of length 0.1 containing a zero. [2 marks]

The continuous polynomial has opposite signs at these endpoints.

Answer: $1.3 < x < 1.4$.

(b) Use the narrower data to give that zero to two decimal places. Explain why the rounding is justified. [3 marks]

A zero lies between 1.32 and 1.325. This entire open interval rounds to 1.32 to two decimal places. Here f is increasing for x>1, so it is the same zero.

Answer: $x \approx 1.32$.

9. A simple flight model has height $h(t) = (t-1)(t-3)(t+2)$ metres and starts at $t = 0$. The model stops at the first ground contact. [5 marks]

(a) Find all algebraic times when the model height is zero. [2 marks]

Set each factor to zero.

Answer: $t = -2, 1, 3$.

(b) Find the first ground-contact time and explain why the other roots do not describe that event. [3 marks]

-2 is before the start; 3 is after the model has already stopped. The initial height is 6 m.

Answer: $t = 1$ second.

10. A least-degree cubic touches the axis at 1, crosses it at -2, and has y-intercept 6. [6 marks]

(a) Find its rule. [4 marks]

Touching needs multiplicity at least 2; crossing needs at least 1. Use $P(0) = 2a = 6$.

Answer: $P(x) = 3(x-1)^2(x+2)$.

(b) Find P(3) and state the end behaviour. [2 marks]

$3(2)^2(5) = 60$; the leading term is $3x^3$.

Answer: $P(3) = 60$; left down/right up.