Real Polynomials - Practice Test 5

Name: _____ Date: _____

10 questions | 49 marks | 75 minutes

Connect exact algebra, restricted domains and numerical evidence without losing solutions.

A scientific calculator may be used.

1. Consider $F(x) = \frac{x^2-4}{x-2}$. [4 marks]

(a) Simplify and state the original domain. [2 marks]

(b) Is F the same function as the polynomial $x + 2$ on all real inputs? [2 marks]

2. $P(x) = 2x^3 - 3x^2 - 11x + 6$. [5 marks]

(a) Show that $\frac{1}{2}$ is a root. [2 marks]

(b) Factor P completely. [3 marks]

3. An odd cubic has rule $P(x) = ax^3 + bx$ and satisfies $P(1) = 2$, $P(2) = 10$. [5 marks]

(a) Find a and b. [3 marks]

(b) Find all real zeros and explain why no others exist. [2 marks]

4. Divide $P(x) = 6x^3 - x^2 - 8x + 4$ by $2x - 1$. [5 marks]

(a) Find Q and R. [3 marks]

(b) Check the remainder by the remainder theorem. [2 marks]

5. $P(2) = 5$ and $P(-2) = 1$. [4 marks]

(a) Find the remainder on division by $x^2 - 4$. [3 marks]

(b) Find the remainder of $P(x) - x - 3$ on the same division. [1 mark]

6. $P(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$. It is known that $(x-2)^2$ is a factor. [5 marks]

(a) Find the remaining quadratic factor. [3 marks]

(b) State the real roots, multiplicities and sign of P elsewhere. [2 marks]

7. Solve $\frac{x^3-x}{x-1}=6$. [5 marks]

(a) State the restriction and form the polynomial equation obtained by multiplication. [2 marks]

(b) Find the allowed solutions. [3 marks]

8. For $f(x)=x^3-x-1$, numerical calculations give $f(1.3)=-0.103$, $f(1.4)=0.344$, $f(1.32)=-0.020032$ and $f(1.325)=0.001203125$. [5 marks]

(a) Give a justified interval of length 0.1 containing a zero. [2 marks]

(b) Use the narrower data to give that zero to two decimal places. Explain why the rounding is justified. [3 marks]

9. A simple flight model has height $h(t) = (t-1)(t-3)(t+2)$ metres and starts at $t = 0$. The model stops at the first ground contact. [5 marks]

(a) Find all algebraic times when the model height is zero. [2 marks]

(b) Find the first ground-contact time and explain why the other roots do not describe that event. [3 marks]

10. A least-degree cubic touches the axis at 1, crosses it at -2, and has y-intercept 6. [6 marks]

(a) Find its rule. [4 marks]

(b) Find P(3) and state the end behaviour. [2 marks]