Real Polynomials - Practice Test 6

Name: _____ Date: _____

10 questions | 58 marks | 75 minutes

Combine coefficient constraints, repeated-root arguments and parameter-dependent real roots.

A scientific calculator may be used.

1. $P_a(x) = (x + a)^3 - x^3$. [3 marks]

(a) Expand P and identify its degree when $a \neq 0$. [2 marks]

(b) What happens when $a = 0$? [1 mark]

2. Consider the monic cubic with factors $(x - a)(x - 2a)(x + 3a)$. [6 marks]

(a) Expand it in terms of a. [3 marks]

(b) Hence find a and all roots if the polynomial equals $x^3 - 28x + 48$. [3 marks]

3. $P(x) = x^3 + ax^2 + bx + 5$ has remainder 3 on division by $x - 1$ and remainder 9 on division by $x + 1$. [6 marks]

(a) Find a and b. [3 marks]

(b) Find the remainder on division by $x^2 - 1$. [3 marks]

4. P is divided by $(x - 1)^2$, leaving remainder $mx + c$. It is known that $P(1) = 3$ and $P'(1) = 4$. [6 marks]

(a) Use the division identity and its derivative to find m and c. [4 marks]

(b) Does this determine the entire polynomial P? [2 marks]

5. $P(x) = x^4 + ax^3 + bx^2 + ax + 1$ has zeros 2 and -1. [7 marks]

(a) Find a and b. [4 marks]

(b) Show that all real roots are $2, \frac{1}{2}, -1$, with -1 repeated. [3 marks]

6. $P_k(x) = x^4 - 2kx^2 + 1$, where k is real. [7 marks]

(a) Solve for x^2 in terms of k. [3 marks]

(b) State the number of distinct real roots for $k < 1$, $k = 1$ and $k > 1$. [4 marks]

7. Consider the equations $x^3 - 3x = c$ for varying real c. [6 marks]

(a) Find the local maximum and minimum of $x^3 - 3x$. [3 marks]

(b) How many distinct real solutions are there when $|c| < 2$, $|c| = 2$, and $|c| > 2$? [3 marks]

8. Let $P(x) = (x + 2)(x - 1)^2$ and $Q(x) = 2P(3 - x)$. [5 marks]

(a) Find the factorised form and roots of Q. [3 marks]

(b) Find its leading coefficient and y-intercept. [2 marks]

9. A sensor calibration is modelled by a polynomial P of degree at most 3. It reports $P(0) = 1$, $P(1) = 2$, $P(2) = 9$, $P(3) = 28$. [6 marks]

(a) Find a polynomial fitting all four values. [4 marks]

(b) Explain why no other polynomial of degree at most 3 can fit the same data. [2 marks]

10. A polynomial P has the form $P(x) = (x - 2)Q(x) + 7$. [6 marks]

(a) Find the remainder of $P(x)^2 - 14P(x)$ on division by $x - 2$. [3 marks]

(b) Prove that $P(x)^2 - 14P(x) + 49$ is divisible by $(x - 2)^2$. [3 marks]