Worked solutions | Algebra

Explore formulas with digital tools

8 questions | 28 marks

Equivalent correct methods and notation earn corresponding marks. Include units and directions where asked. Each worked step identifies the mark allocation.

Cost: $C = pn + f$. Here C is total dollars, p is dollars per visit, n is visits, and f is a fee paid once. Rectangle perimeter: $P = 2l + 2w$, with P, length l and width w in centimetres.

Written companion: calculate and record each comparison here. Complete the digital Formula lab in the lesson separately; this sheet alone does not demonstrate digital-tool use.

1. For $C = pn + f$, p is the price per visit, n is visits and f is a fixed fee paid once. Set $p = 3$, $n = 4$, $f = 5$. Show a substitution that evaluates the formula and state the total in dollars. [2 marks]

$C = 3 \times 4 + 5$ (1 mark for correct substitution).

$C = 12 + 5 = 17$, so the total is 17 dollars (1 mark).

Answer: 17 dollars.

2. In the cost formula $C = pn + f$, hold $p = 5$ and $f = 3$ fixed. Change n through $2, 3, 4$. Record a comparison table showing every input and output, then describe the effect of each extra visit. [4 marks]

$5 \times 2 + 3 = 13$; record $(5, 2, 3, 13)$ (1 mark).

$5 \times 3 + 3 = 18$; record $(5, 3, 3, 18)$ (1 mark).

$5 \times 4 + 3 = 23$; record $(5, 4, 3, 23)$ (1 mark).

$18 - 13 = 5$ and $23 - 18 = 5$: each extra visit adds 5 dollars while price and fee stay fixed (1 mark).

Answer: Rows (p, n, f, C): $(5, 2, 3, 13)$, $(5, 3, 3, 18)$, $(5, 4, 3, 23)$. Each extra visit adds 5 dollars.

3. Use $C = pn + f$ with $n = 3$ and $f = 4$ fixed. Vary p through $2, 4, 6$ dollars per visit. Record each total and describe the cost change for each 2-dollar rise in price per visit. [4 marks]

$C = 2 \times 3 + 4 = 10$ (1 mark).

$C = 4 \times 3 + 4 = 16$ (1 mark).

$C = 6 \times 3 + 4 = 22$ (1 mark).

$16 - 10 = 6$ and $22 - 16 = 6$: the 2-dollar increase applies to each of the 3 visits, giving $2 \times 3 = 6$ dollars (1 mark).

Answer: The totals are 10, 16 and 22 dollars. Each 2-dollar rise in price per visit adds 6 dollars to the total.

4. Keep $p = 4$ and $n = 3$ in $C = pn + f$. Compare $f = 1, 4, 7$. State the three totals, their successive differences, and why this investigation isolates the fixed fee's effect. [3 marks]

$4 \times 3 + 1 = 13$, $4 \times 3 + 4 = 16$, $4 \times 3 + 7 = 19$ (1 mark for all three totals).

$16 - 13 = 3$ and $19 - 16 = 3$ dollars (1 mark for both differences).

Price and visits stay fixed, so only the fixed fee can cause the change in this formula (1 mark).

Answer: The totals are 13, 16 and 19 dollars, with successive increases of 3 dollars. Only the fixed fee changes; price and visits are held fixed.

5. Use $P = 2l + 2w$. Hold $w = 4$ cm fixed and vary l through $3, 5, 7$ cm. Record a table of lengths and perimeters, state the fixed width, and describe the change in perimeter for each 2 cm increase in length. [4 marks]

For $l = 3$: $P = 2 \times 3 + 2 \times 4 = 14$ cm (1 mark).

For $l = 5$: $P = 2 \times 5 + 2 \times 4 = 18$ cm (1 mark).

For $l = 7$: $P = 2 \times 7 + 2 \times 4 = 22$ cm (1 mark).

Width stays 4 cm; $18 - 14 = 4$ and $22 - 18 = 4$, so each 2 cm increase in length adds 4 cm to perimeter (1 mark).

Answer: With width 4 cm, $(l, P) = (3, 14), (5, 18), (7, 22)$ in centimetres. Each 2 cm increase in length adds 4 cm to perimeter.

6. A student predicts: 'Doubling a rectangle's length while keeping its width fixed always doubles its perimeter.' Using $P = 2l + 2w$, keep $w = 3$ cm and compare $l = 4$ cm with $l = 8$ cm. Use the two outputs to decide whether the prediction is correct. [3 marks]

At $l = 4$, $P = 2 \times 4 + 2 \times 3 = 14$ cm (1 mark).

At $l = 8$, $P = 2 \times 8 + 2 \times 3 = 22$ cm (1 mark).

$2 \times 14 = 28$, not 22. The unchanged width contribution prevents the perimeter from doubling in this example (1 mark).

Answer: The prediction is incorrect. The perimeters are 14 cm and 22 cm; double 14 cm would be 28 cm.

7. For $C = pn + f$, keep $p = 3$ and $f = 5$. Predict whether changing n from 2 to 4 doubles the total cost. Calculate both settings, record the totals and explain the result using the fixed fee. [4 marks]

Record a prediction with a reason before testing (1 process mark; an initial prediction may be revised after testing).

At $n = 2$, $C = 3 \times 2 + 5 = 11$ dollars (1 mark).

At $n = 4$, $C = 3 \times 4 + 5 = 17$ dollars (1 mark).

Double 11 is 22, not 17. The visit part doubles from 6 to 12 dollars, but the fee stays 5 dollars, so the total increases by 6 dollars (1 mark for the checked conclusion and explanation).

Answer: The total does not double: it changes from 11 dollars to 17 dollars. The 5-dollar fixed fee is paid once in each total.

8. A student records $(p, n, f) = (2, 3, 4)$ giving $C = 10$, then $(3, 5, 4)$ giving $C = 19$. They conclude: 'Raising price per visit by 1 dollar raises total cost by 9 dollars.' Explain the problem. Design and calculate a fair comparison using $n = 3$ and $f = 4$, then state the price change's effect. [4 marks]

The first comparison changes p and n together (1 mark).

Hold $n = 3$ and $f = 4$ fixed and vary only p from 2 to 3 (1 mark).

Record $C = 2 \times 3 + 4 = 10$ and $C = 3 \times 3 + 4 = 13$ dollars (1 mark for both totals).

$13 - 10 = 3$: a 1-dollar rise per visit adds 3 dollars across 3 visits (1 mark).

Answer: Both price and visits changed, so the 9-dollar difference does not isolate price. With $n = 3$ and $f = 4$, changing p from 2 to 3 changes cost from 10 to 13 dollars: a 3-dollar increase.

Curriculum alignment

Australian Curriculum, verified against QCAA | 9.0 | AC9M7A06

Use the on-page Formula lab for controlled investigations of cost and perimeter formulas. Record inputs, outputs and changes, then explain the relationships. The digital activity and written record work together; the worksheet alone does not demonstrate use of a digital tool. Rearranging formulas is outside this lesson.

Official curriculum source | Checked 2026-11-02