Quadratic models and applications

6 questions | 24 marks

Award the shown method marks for equivalent correct working.

1. A rectangle is x m wide and $(x+2)$ m long. Its area is 48 m^2. Form an equation and find its dimensions. [4 marks]

Use $x(x+2)=48$ [1].
Factorise $x^2+2x-48=(x+8)(x-6)$ [1].
Give roots $-8, 6$ and reject the negative width [1].
State dimensions 6 m by 8 m [1].
Answer: $x^2+2x-48=0$. The rectangle is 6 m by 8 m.

2. A square picture has side length x cm. A frame is 2 cm wide all the way around it. The picture and frame together form a square of area 81 cm^2. Find x. [4 marks]

The outside side length is $x+4$ [1].
Use $(x+4)^2=81$ [1].
Find $x+4=\pm 9$, so $x=5$ or $x=-13$ [1].
Reject the negative side length and give 5 cm [1].
Answer: $x=5$ cm.

3. A discount plan charges $(12-n)$ dollars per item when n items are bought. Here n is a whole number from 1 to 11. The total price is $R=n(12-n)$. [4 marks]

(a) Find R for $n=4$.

Substitute $n=4$ and find $4(8)=32$ dollars [1].
Factorise to $(n-5)(n-7)=0$ [1].
Give both allowed values $5, 7$ [1].
Answer: 32 dollars.

(b) Find both values of n for which $R=35$.

Form $n^2-12n+35=0$ [1].
Answer: $n=5$ or $n=7$.

4. A rectangle has perimeter 24 m and area 32 m². Let its width be w m. [4 marks]

(a) Show that its length is $(12 - w)$ m.

From $2l + 2w = 24$, get $l + w = 12$ and $l = 12 - w$ [1].

Answer: $l = 12 - w$.

(b) Find both roots of $w(12 - w) = 32$.

Factorise $w^2 - 12w + 32 = (w - 4)(w - 8)$ [1]; give $4, 8$ [1].

Answer: $w = 4$ or $w = 8$.

(c) State the rectangle's dimensions.

Both choices describe a 4 m by 8 m rectangle [1].

Answer: 4 m by 8 m.

5. A height model is $h = t(8 - t)$ metres for $0 \le t \le 8$ seconds. [4 marks]

(a) Find both times when $h = 12$ m.

Rearrange to $t^2 - 8t + 12 = 0$ [1].

Factorise to $(t - 2)(t - 6) = 0$ [1]; give both allowed times $2, 6$ seconds [1].

Answer: 2 seconds and 6 seconds.

(b) Explain why two times are possible in this model.

Explain the upward and downward parts of the motion [1].

Answer: It reaches 12 m once on the way up and once on the way down.

6. One model is linear and the other is quadratic. For inputs $x = 0, 1, 2, 3$, model A gives $1, 4, 9, 16$ and model B gives $2, 5, 8, 11$. [4 marks]

(a) Which is quadratic? Explain using differences.

Find first differences $3, 5, 7$ for A [1]; constant second differences $2, 2$ identify it as quadratic [1].

Answer: Model A is quadratic: first differences $3, 5, 7$ give second differences $2, 2$. Model B is linear.

(b) Predict each model's output for $x = 4$.

Predict A gives 25 [1] and B gives 14 [1].

Answer: A: 25; B: 14.