Year 9 Quadratics and Parabolas - Applications Test

12 questions | 50 marks

Award the shown method marks for equivalent correct working.

1. [4 marks]

(a) Expand $(x+3)(x+5)$.

Multiply each term: $x^2+5x+3x+15$. (1 mark)

Collect like terms to get $x^2+8x+15$. (1 mark)

Answer: $x^2+8x+15$.

(b) Factorise $x^2+7x+12$.

Find two numbers with product 12 and sum 7: 3 and 4. (1 mark)

Write $(x+3)(x+4)$. (1 mark)

Answer: $(x+3)(x+4)$.

2. Solve $x^2-6x=0$. Give both solutions. [3 marks]

Factorise: $x(x-6)=0$. (1 mark)

The first factor gives $x=0$. (1 mark)

The second factor gives $x=6$. Do not divide by x, because that would lose the zero solution. (1 mark)

Answer: $x=0$ or $x=6$.

3. A rectangular garden is x metres wide and $(x + 3)$ metres long. Its area is 40 square metres. [4 marks]

(a) Write an equation for the area.

Area is width times length, so $x(x + 3) = 40$. (1 mark)

Answer: $x(x + 3) = 40$.

(b) Find the width and length. Explain why you reject one solution.

Rearrange to $x^2 + 3x - 40 = 0$ and factorise as $(x + 8)(x - 5) = 0$. (1 mark)

The possible values are $x = -8$ and $x = 5$; reject -8 because a garden cannot have negative width. (1 mark)

Width 5 m and length $5 + 3 = 8$ m; check $5 \times 8 = 40$. (1 mark)

Answer: The width is 5 m and the length is 8 m. Reject $x = -8$ because a length must be positive.

4. A photo is 8 cm wide and 4 cm high. A border of width x cm is added on every side. The photo and border together have area 96 square centimetres. [4 marks]

Outside dimensions of the photo and border



(a) Write an equation using the outside dimensions.

Each outside dimension includes two borders, so $(8 + 2x)(4 + 2x) = 96$. (1 mark)

Answer: $(8 + 2x)(4 + 2x) = 96$.

(b) Show that it simplifies to $x^2 + 6x - 16 = 0$.

Expand to $4x^2 + 24x + 32 = 96$, then subtract 96 and divide by 4 to get $x^2 + 6x - 16 = 0$. (1 mark)

Answer: $x^2 + 6x - 16 = 0$.

(c) Find the border width.

Factorise: $(x + 8)(x - 2) = 0$, giving $x = -8$ or $x = 2$. (1 mark)

Reject the negative width, so the border is 2 cm wide. The outside area is $12 \times 8 = 96$ square centimetres. (1 mark)

Answer: 2 cm.

5. A gardener uses 20 m of fencing for three sides of a rectangle. A wall forms the fourth side. Each side at right angles to the wall has length x metres. [5 marks]

(a) Write the length of the fenced side parallel to the wall.

Two sides use $2x$ metres, leaving $20 - 2x$ metres. (1 mark)

Answer: $20 - 2x$ metres.

(b) Write the area A in terms of x.

$A = x(20 - 2x)$. (1 mark)

Answer: $A = x(20 - 2x)$.

(c) The same area rule can be written $A = 50 - 2(x - 5)^2$. Use this form to find the largest possible area and the rectangle's dimensions. Explain why it is the largest.

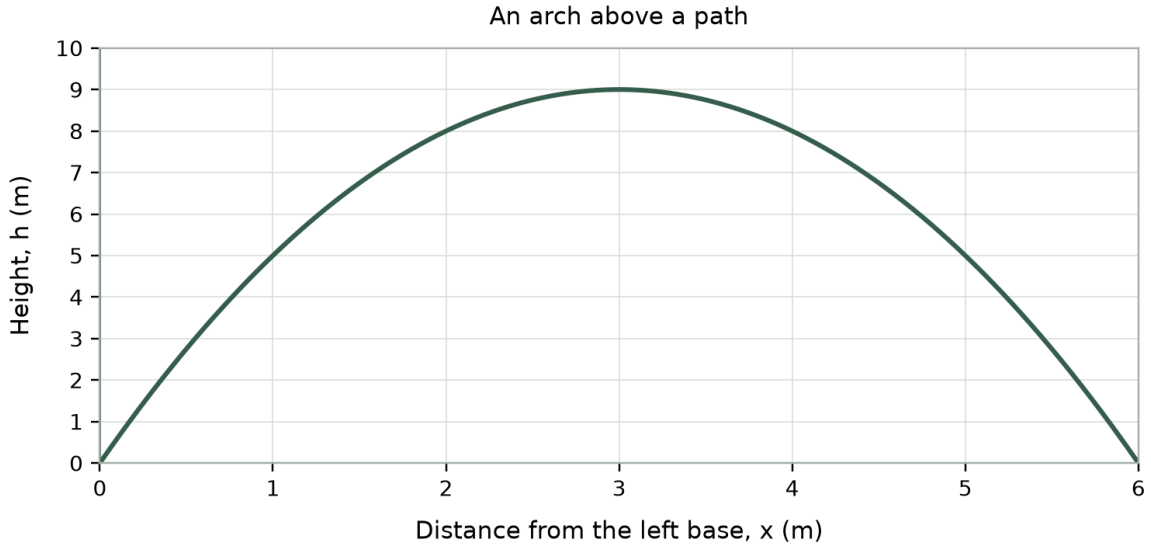
The square $(x - 5)^2$ is at least zero, so $50 - 2(x - 5)^2$ cannot exceed 50. (1 mark)

The largest area is 50 square metres when $x = 5$. (1 mark)

The other dimension is $20 - 2(5) = 10$ m, giving a 5 m by 10 m rectangle. (1 mark)

Answer: The largest area is 50 square metres, with width 5 m and length 10 m. The squared term cannot be negative.

6. The graph shows an arch. Its height is $h = -x^2 + 6x$, where x is the distance from its left base. Both distances are in metres. [5 marks]



(a) Read the width of the arch at ground level.

The ground-level points are $x = 0$ and $x = 6$, so the width is 6 m. (1 mark)

Answer: 6 m.

(b) State its greatest height and the value of x there.

The top of the curve has height 9 m. (1 mark)

This occurs at $x = 3$ m. (1 mark)

Answer: The greatest height is 9 m, at $x = 3$ m.

(c) Use the rule to find both places where the arch is 8 m high.

Set $-x^2 + 6x = 8$, so $x^2 - 6x + 8 = 0$ and $(x - 2)(x - 4) = 0$. (1 mark)

The arch is 8 m high at $x = 2$ m and $x = 4$ m. (1 mark)

Answer: At $x = 2$ m and $x = 4$ m.

7. Use $y = x^2 - 4x + 3$. [5 marks]

(a) Make a table of values for $x = 0, 1, 2, 3, 4$.

Substitute to obtain $(0, 3)$, $(1, 0)$, $(2, -1)$, $(3, 0)$ and $(4, 3)$. (1 mark)

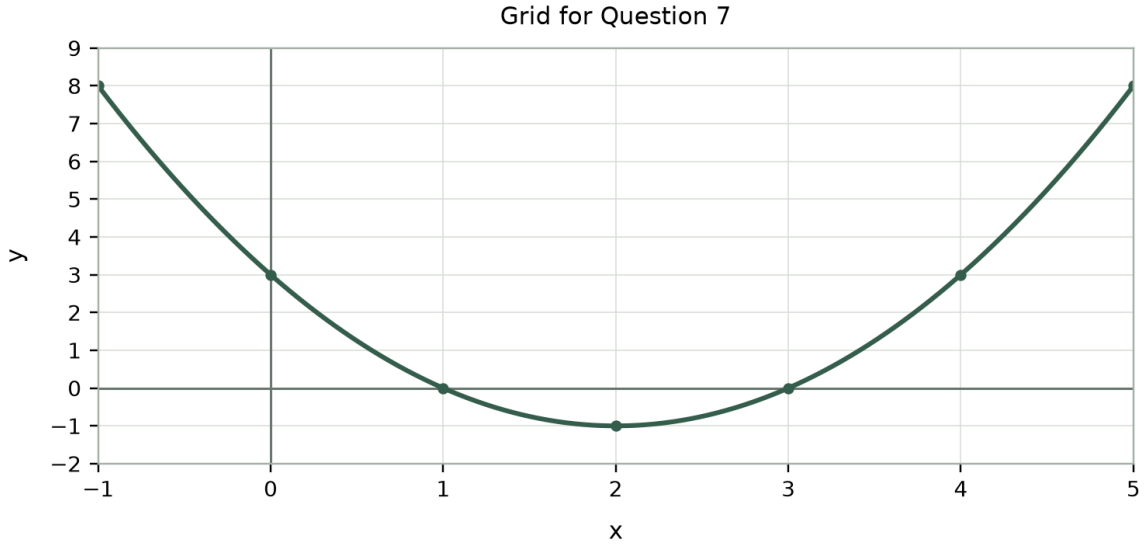
Answer: The y values are $3, 0, -1, 0, 3$.

(b) Plot the points on the grid and join them with a smooth curve.

Plot all five points correctly using the labelled grid. (1 mark)

Join them with a smooth upward-opening curve, not straight line segments. (1 mark)

Answer: An upward-opening parabola through the five points.



(c) State the turning point and the axis of symmetry.

The turning point is $(2, -1)$ and the axis is $x = 2$. (1 mark)

Answer: Turning point $(2, -1)$; axis $x = 2$.

(d) Use your graph to solve $x^2 - 4x + 3 = 0$.

The curve crosses the x-axis at $x = 1$ and $x = 3$. (1 mark)

Answer: $x = 1$ or $x = 3$.

8. A pattern has these values: $n = 0, 1, 2, 3$ give $T = 2, 5, 10, 17$. Two rules are suggested: $T = 3n + 2$ and $T = n^2 + 2n + 2$. [4 marks]

(a) Which rule fits all four values? Show a calculation that distinguishes the rules.

Choose $T = n^2 + 2n + 2$, which gives $2, 5, 10, 17$ for the four inputs. (1 mark)

For example, at $n = 2$, the quadratic gives $4 + 4 + 2 = 10$, but the linear rule gives $6 + 2 = 8$. (1 mark)

Answer: $T = n^2 + 2n + 2$. For $n = 2$, it gives 10, while the linear rule gives 8.

(b) Use that rule to find T when $n = 4$.

At $n = 4$, $T = 16 + 8 + 2 = 26$. (1 mark)

Answer: 26.

(c) Explain why checking only $n = 0$ and $n = 1$ would not settle which rule to use.

Both rules match the first two values, so those checks alone cannot distinguish them. (1 mark)

Answer: Both rules give 2 and 5 at the first two inputs.

9. A stall's profit is modelled by $P = -p^2 + 10p - 16$, where p is the ticket price in dollars and P is profit in dollars. Use prices from 0 to 10 dollars. [4 marks]

(a) Find the two prices that give zero profit.

Set $P = 0$, multiply by -1 and factorise: $p^2 - 10p + 16 = (p - 2)(p - 8) = 0$. (1 mark)

The zero-profit prices are 2 dollars and 8 dollars. (1 mark)

Answer: 2 dollars and 8 dollars.

(b) Find the profit when the price is 4 dollars.

$P = -4^2 + 10(4) - 16 = -16 + 40 - 16 = 8$ dollars. (1 mark)

Answer: 8 dollars profit.

(c) Is a price of 9 dollars profitable? Show a calculation.

At $p = 9$, $P = -81 + 90 - 16 = -7$, so the model gives a 7 dollar loss. (1 mark)

Answer: No: $P = -7$, a loss of 7 dollars.

10. Mia solves $(x-4)(x+3)=0$ and writes $x=4$ or $x=3$. [4 marks]

(a) Correct her solutions and explain the sign error.

The first factor gives $x=4$. (1 mark)

The second factor gives $x=-3$, because subtracting 3 from both sides of $x+3=0$ gives $x=-3$. (1 mark)

Answer: $x=4$ or $x=-3$; $x+3=0$ gives -3, not 3.

(b) If x is a length in metres, which solution is allowed? Explain why.

Only $x=4$ m is allowed for a positive length; reject -3 m. (1 mark)

Answer: Only 4 m is allowed because a length must be positive.

(c) Check $x=3$ in the original equation to show why it is not a solution.

Substitution gives $(3-4)(3+3)=(-1)(6)=-6$, which is not zero. (1 mark)

Answer: $(3-4)(3+3)=-6$, not zero.

11. A monic quadratic has x-intercepts at $x=-2$ and $x=5$. Here, monic means that the coefficient of x^2 is 1. [4 marks]

(a) Write its rule in factorised form.

The intercepts give $y=(x+2)(x-5)$, with leading coefficient 1. (1 mark)

Answer: $y=(x+2)(x-5)$.

(b) Expand your rule.

Expand to $y=x^2-5x+2x-10=x^2-3x-10$. (1 mark)

Answer: $y=x^2-3x-10$.

(c) State the y-intercept.

Set $x=0$ to get the y-intercept $(0,-10)$. (1 mark)

Answer: $(0,-10)$.

(d) State the axis of symmetry.

The axis lies halfway between the roots: $x=(-2+5)/2=1.5$. (1 mark)

Answer: $x=1.5$.

12. Two curves have rules $y = 9 - (x - 3)^2$ and $y = 9 - 2(x - 3)^2$. [4 marks]

(a) State the turning point shared by both curves.

Both squared terms are zero at $x = 3$, so both turning points are $(3, 9)$. (1 mark)

Answer: $(3, 9)$.

(b) Find each y value at $x = 2$.

At $x = 2$, the first curve gives $9 - 1 = 8$ and the second gives $9 - 2 = 7$. (1 mark)

Answer: 8 for the first curve and 7 for the second.

(c) Which curve is narrower? Use your values to explain.

The second curve is narrower: at the same horizontal distance from the turning point, it drops farther. (1 mark)

Answer: $y = 9 - 2(x - 3)^2$ is narrower.

(d) The first curve is moved up 2 units. Write its new rule.

Add 2 to the first rule: $y = 11 - (x - 3)^2$. (1 mark)

Answer: $y = 11 - (x - 3)^2$.